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GENERALIZED DIFFUSION THEORY OF MULTITEMPERATURE HOMOGENEOUS MIXTURES

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A generalized diffusion theory is constructed for motion of homogeneous mixtures consisting of components at different temperatures.

The theory of motion of mutually penetrable media, e.g., the motion of various materials and phases with consideration of phase transitions, chemical reactions, heat transfer, and radiation; the motion of liquids and gases with various particles; and the motion of liquids and gases in soils, is important in solution of numerous practical problems, in particular, those related to chemical production, hydroenergetics, hydromelioration, and the development of the petroleum and gas industry.

There exist various approaches to the problem of study of the complex motion and varied processes in interpenetrating media. One of these is diffusion theory.

We construct below a generalized diffusion theory of motion of multitemperature homogeneous mixtures. In contrast to classical diffusion theory [1-4] here we introduce a certain general characteristic velocity of the mixture as a whole, and diffusion flows are defined relative to this velocity. Using general integral laws of the mechanics of mixtures, a new system of equations is obtained for determination of the unknown values, generalizing Prigogine's theorem [2, 3]. This system permits transforming from one characteristic velocity to another in the general case of motion of different-temperature homogeneous mixtures. It is shown that with consideration of the contribution of diffusion flows to the energy of the total mixture the equations for determination of the diffusion flows are differential, and not algebraic, equations. The equations for change in component concentration are of the hyperbolic type, not parabolic, as in classical theory, i.e., generalized diffusion laws are obtained, as were proposed in [5, 6] in analogy to the rheology of viscoelastic media.

1. Basic Integral Laws of the Mechanics of Mixtures. We will consider the motion of mixtures consisting of n components. We will assume that all these components fill one and the same volume, occupied by the mixture. Let ρ_k, u_k be the density and velocity of the k-th component. It is known that in the theory of homogeneous mixtures, together with the density ρ_k it is necessary to consider other quantities characterizing the inertness of each component. These quantities are M_k, the molar mass; V_k, the molar partial volume; and N_k, the number of moles of the k-th component per unit volume of mixture.

Let u_a be some characteristic velocity of the mixture. We will assume that u_a may be expressed as a linear combination of u_k with the aid of some system of normalizing weights a_k, i.e.,

$$u_a = \sum_{k=1}^{n} a_k u_k; \quad \sum_{k=1}^{n} a_k = 1, \tag{1.1}$$

where a_k may depend only on ρ_k, M_k, N_k and V_k.

The motion of the k-th component relative to an observer moving with a velocity u_a is determined with the aid of a generalized diffusion current J_k^a, equal to

$$J_k^a = \rho_k (u_k - u_a); \quad \sum_{k=1}^{n} \frac{a_k}{\rho_k} J_k^a = 0. \tag{1.2}$$

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In order to obtain equations for the determination of u_a, J_k^a and other mixture parameters it is necessary to postulate laws for the mechanics of continuous media. In integral form these appear as

$$
\begin{gathered}
\frac{d}{dt}\int\limits_V \sum_{k=1}^{n} \rho_k dV = \int\limits_V \frac{\partial}{\partial t}\left(\sum_{k=1}^{n}\rho_k\right) dV + \int\limits_S n \sum_{k=1}^{n} \rho_k u_k dS = 0;\\
\frac{d}{dt}\int\limits_V \sum_{k=1}^{n} \rho_k u_k dV = \int\limits_V \frac{\partial}{\partial t}\left(\sum_{k=1}^{n}\rho_k u_k\right) dV + \int\limits_S n\left(\sum_{k=1}^{n}\rho_k u_k u_k\right) dS = \int\limits_V \sum_{k=1}^{n}\rho_k f_k dV + \int\limits_S n\cdot P dS;\\
\frac{d}{dt}\int\limits_V \sum_{k=1}^{n} (r_k \times \rho_k u_k)\, dV = \int\limits_V \sum_{k=1}^{n} r_k \times \rho_k f_k dV + \int\limits_S r\times(nP)\, dS;\\
\frac{d}{dt}\int\limits_V \sum_{k=1}^{n}\left(\frac{1}{2}\rho_k u_k^2 + \rho\varepsilon\right) dV = \int\limits_V \frac{\partial}{\partial t}\left(\sum_{k=1}^{n}\frac{1}{2}\rho_k u_k^2 + \rho\varepsilon\right) dV +\\
+\int\limits_S n\sum_{k=1}^{n}\left(\frac{1}{2}\rho_k u_k^2 + \rho_k\varepsilon\right) u_k dS = \int\limits_V \rho h^* dV + \int\limits_V \sum_{k=1}^{n}(\rho_k f_k u_k)\, dV -\\
-\int\limits_S n\cdot q dS + \int\limits_S n\,(P\cdot u_a)\, dS + \int\limits_S n\left(\sum_{k=1}^{n} R_k^a \cdot \frac{J_k^n}{\rho_k}\right) dS.
\end{gathered}
\tag{1.3}
$$

In Eq. (1.3) n is the unit vector along the external normal of the surface S; dV and dS are the elements of the volume V and the surrounding surface S; f_k, the density of the external force acting on the k-th continuum; P, the stress tensor; r_k, the radius of the center of mass of the k-th continuum located within the element dV; r, the radius of the center of mass of the mixture as a whole (at a given moment of time t it is assumed that $r_k = r$); ε is the internal energy of a unit mass of mixture; R_k^a are second range tensors defining additional surface interactions in the mixture; h*, the intensity of a heat source; q, the thermal flux; the operation ∂/∂t (. . .) indicates partial differentiation with respect to time; ∇, the derivative along the coordinate; · indicates the scalar product; × indicates the cross product of adjacent indices of the tensor quantities. Moreover, it is assumed in Eq. (1.3) that the stress tensor P operates only on the general characteristic velocity of the mixture u_a, while the tensors R_k^a operate on the relative velocities of the continua.

2. Basic Differential Equations of Mixture Motion. We will employ the defining equations (1.1) and (1.2) to transform Eq. (1.3) to the following differential form:

$$
\begin{gathered}
\frac{\partial\rho_k}{\partial t} + \nabla\cdot(\rho_k u_a) = -\nabla\cdot J_k^a + m_k; \quad \sum_{k=1}^{n} m_k = 0;\\
\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\cdot u_a) = -\sum_{k=1}^{n}\nabla\cdot J_k^a; \quad \sum_{k=1}^{n}\rho_k = \rho; \quad \rho f = \sum_{k=1}^{n}\rho_k f_k;\\
\rho\,\frac{d^{(a)} u_a}{dt} = \rho f - \nabla\cdot P - \sum_{k=1}^{n}\left(\frac{D^{(a)} J_k^a}{Dt} + \nabla\cdot\frac{J_k^a J_k^a}{\rho_k}\right); \quad P^A = 0;\\
\rho\,\frac{d^{(a)}\varepsilon}{dt} = \rho h^* - \nabla\cdot q + \sum_{k=1}^{n}\frac{1}{2} m_k\left(\frac{J_k^a}{\rho_k}\right)^2 + \sum_{k=1}^{n} J_k^a\cdot\Bigg\{f_k - \nabla\varepsilon -\\
-\frac{d^{(a)} u_a}{dt} - \frac{Q_k^a}{\rho_k}\Bigg\} + P\cdot\cdot\nabla u_a + \sum_{k=1}^{n} R_k^a\cdot\cdot\nabla\frac{J_k^a}{\rho_k}.
\end{gathered}
\tag{2.1}
$$

In Eq. (2.1) m_k indicates the change in mass of the k-th continuum due to physicochemical transformations; P^A, the antisymmetric portion of the stress tensor; the operation (. .) denotes convolution over all two indices; $d^{(a)}/dt$, the complete time derivative; $D^{(a)}/Dt$, a derivative defining the change in time with respect to a generalized Lagrangian coordinate system moving and deforming with a velocity u_a [7], these derivatives having the form

$$\frac{d^{(a)}}{dt}(\cdots) = \frac{\partial}{\partial t}(\cdots) + (u_a \cdot \nabla)(\cdots); \qquad (2.2)$$

$$\frac{D^{(a)}}{Dt}(\cdots) = \frac{d^{(a)}}{dt}(\cdots) + [(\cdots)\cdot\nabla]\, u_a + (\cdots)(\nabla \cdot u_a).$$

Moreover, in Eq. (2.1) the new quantities Q_k^a were introduced, defined in the following manner:

$$\frac{D^{(a)} J_k^a}{Dt} + Q_k^a + \nabla\cdot\left(R_k^a - \frac{J_k^a J_k^a}{\rho_k}\right). \qquad (2.3)$$

It is easy to demonstrate that if we assume

$$\sum_{k=1}^{n} \frac{a_k}{\rho_k} R_k^a = 0, \qquad (2.4)$$

then Q_k^a must satisfy the relationship

$$\sum_{k=1}^{n} \frac{a_k}{\rho_k} Q_k^a = \sum_{k=1}^{n} \left\{ \frac{a_k}{\rho_k} \nabla\cdot\left(\frac{J_k^a J_k^a}{\rho_k} - R_k^a\right) - J_k^a \frac{d^{(a)}}{dt}\left(\frac{a_k}{\rho_k}\right)\right\}. \qquad (2.5)$$

We note that Eqs. (2.1) and (2.3) are exact equations for determination of the change in continuum density, characteristic velocity u_a, internal energy ε, and diffusion flows J_k^a. They coincide with the well-known equations of classical diffusion theory [1-2] if we choose as the characteristic general velocity the mean mass velocity, and the terms $D^{(a)}J_k^a/Dt$, $\nabla J_k^a J_k^a/\rho_k$ and R_k^a can be neglected. Together with specifying the concrete form of the internal energy and the dissipation function, these equations can serve as a basis for description of the motion of various mutually penetrating media, not only homogeneous and heterogeneous mixtures, but also liquids and gases in soil and rock.

In the theory of homogeneous mixtures one of the following characteristic velocities is often employed: mean mass, molar, volume, or the velocity of one of the components. The advantages of each of these were discussed in [3, 10]. To describe the general motion of heterogeneous mixtures the mean mass, or mean volume velocity, or simply the velocity of the carrier phase is used. The theory of drift flow, based on use of the mean volume velocity of the mixture, is often found in studies of many types of flows in gas–liquid systems, and also in the flows of suspensions of solid particles in liquids of the fluidized type [8]. Use of the mean volume velocity is still more convenient if the true densities of the phases are constant, since in this case its divergence is equal to zero, while at the same time the other characteristic velocities do not conform to this important mathematical limitation. Moreover, with the aid of the mean volume velocity the coefficient of effective mixture viscosity may be calculated from the phase viscosity coefficients and the volume concentration of the discrete phases [9].

We note that the differential equations (2.3) for determination of the diffusion flows J_k^a contain not only the classical Fick diffusion law, the laws of thermodiffusion, barodiffusion, sedimentation, and Darcy filtration, but also their generalizations with time relaxation, as proposed in [5, 6] in analogy to the rheology of viscoelastic media. Moreover, the presence of the terms $D^{(a)}J_k^a/Dt$ in the equations for the determination of u_a permit a description of other time effects (creep, stress relaxation) in interpenetrating media.

3. Generalized Diffusion Theory of Different-Temperature Homogeneous Mixtures.

Let there be a homogeneous mixture consisting of n components with differing temperatures T_k. For simplicity the absence of chemical reactions will be assumed. We will assume that ε is the internal energy of a unit mass of mixture,

$$\varepsilon = \varepsilon(\rho, c_k, s_k); \quad c_k = \rho_k/\rho, \qquad (3.1)$$

where s_k is the entropy of the k-th component per unit mass of mixture.

Aside from ε we introduce other thermodynamic functions of the multitemperature mixture: F, free energy; h, enthalpy; Ψ, thermodynamic potential. These are related to each other as follows [1]:

$$F = \varepsilon - \sum_{k=1}^{n} T_k s_k = F(\rho, c_k, T_k);$$

$$h = \varepsilon + \frac{p}{\rho} = h(p,\ c_k,\ s_k);$$
$$\Psi = \varepsilon - \sum_{k=1}^{n} T_k s_k + \frac{p}{\rho} = \Psi(p,\ c_k,\ T_k). \tag{3.2}$$

In Eq. (3.2) p is the pressure of the mixture; ρ, total density; c_k, mass concentration of the k-th component. From Eq. (3.2) it is simple to obtain

$$p = \rho^2 \frac{\partial\varepsilon}{\partial\rho} = \rho^2 \frac{\partial F}{\partial\rho};\quad \mu_k = \frac{\partial\varepsilon}{\partial c_k} = \frac{\partial F}{\partial c_k} = \frac{\partial h}{\partial c_k} = \frac{\partial\Psi}{\partial c_k};$$
$$T_k = \frac{\partial\varepsilon}{\partial s_k} = \frac{\partial h}{\partial s_k};\quad s_k = -\frac{\partial F}{\partial T_k} = -\frac{\partial\Psi}{\partial T_k};\quad \frac{1}{\rho} = \frac{\partial h}{\partial p} = \frac{\partial\Psi}{\partial p}, \tag{3.3}$$

where μ_k is the chemical potential of the k-th component.

We assume that for multitemperature homogeneous mixtures all the thermodynamic functions are homogeneous in the first power of the functions in mass, i.e., they may be represented in the form

$$\Psi = \sum_{k=1}^{n} c_k \mu_k. \tag{3.4}$$

It is simple to demonstrate that from Eqs. (3.2)-(3.4) there follow a generalized Gibbs thermodynamic equation and additional relationships:

$$\sum_{k=1}^{n} c_k \nabla\mu_k = \frac{1}{\rho}\nabla p - \sum_{k=1}^{n} s_k \nabla T_k;$$
$$\sum_{k=1}^{n} c_k (\nabla\mu_k)_{p,T_k} = 0;\quad \frac{\partial\mu_k}{\partial p} = v_k;\quad s_k = -c_k \frac{\partial\mu_k}{\partial T_k}, \tag{3.5}$$

where v_k is the specific partial volume of the k-th component.

With consideration of Eq. (3.5) from the last equation of (2.1) we obtain

$$\rho \sum_{k=1}^{n} T_k \frac{d^{(a)}s_k}{dt} = \rho h^* - \nabla\cdot q + \nabla\cdot\left\{\sum_{k=1}^{n} J_k^a \left(\frac{p}{\rho} - \sum_{i=1}^{n} c_i\mu_i\right)\right\} +$$
$$+ \sum_{k=1}^{n} \mu_k \nabla\cdot J_k^a + \sum_{k=1}^{n} J_k^a \cdot \left\{f_k - \frac{d^{(a)}u_a}{dt} - \frac{Q_k^a}{\rho_k}\right\} -$$
$$- \sum_{k=1}^{n}\sum_{i=1}^{n} J_i^a \cdot (T_k \nabla s_k + s_k \nabla T_k) + \tau_a \cdot\cdot e_a + \sum_{k=1}^{n} R_k^a \cdot\cdot \nabla \frac{J_k^a}{\rho_k}. \tag{3.6}$$

In Eq. (3.6) τ_a is the viscous stress tensor and e_a is the deformation velocity tensor. These tensors have the form

$$\tau_a = pI + P;\quad e_a = \frac{1}{2}[(\nabla u_a) + (\nabla u_a)^T]. \tag{3.7}$$

Equation (3.6) still does not permit a description of the second law of the thermodynamics of irreversible processes, since it is formulated only for the entropy of the mixture as a whole. To achieve this description we will assume that from Eq. (3.6) we may postulate the following equations for change in entropy s_k of each component:

$$\rho \frac{d^{(a)}s_k}{dt} + \sum_{i=1}^{n} J_i^a \cdot \nabla s_k = \rho_k \frac{h_k^*}{T_k} - \frac{1}{T_k}\nabla\cdot q_k + \frac{1}{T_k}\nabla\cdot\left\{J_k^a\left(\frac{p}{\rho} - \sum_{i=1}^{n} c_i\mu_i\right)\right\} + \frac{\mu_k}{T_k}\nabla\cdot J_k^a +$$

$$+\frac{1}{T_k} J_k^a \cdot \left\{f_k - \frac{d^{(a)} u_a}{dt} - \frac{Q_k^a}{\rho_k}\right\} - \sum_{i=1}^{n} J_i^a \cdot s_k \frac{\nabla T_k}{T_k} + \frac{1}{T_k} R_k^a \cdot\cdot \nabla \frac{J_k^a}{\rho_k} + \frac{X_k}{T_k}. \tag{3.8}$$

In Eq. (3.8) the new quantities: h_k^*, a heat source; q_k, the thermal flux toward the k-th component; and X_k, an additional term appearing as a result of division of general equation (3.6), were introduced. These quantities must satisfy the following conditions:

$$\sum_{k=1}^{n} \rho_k h_k^* = \rho h; \quad \sum_{k=1}^{n} q_k = q; \quad \sum_{k=1}^{n} X_k = \tau_a \cdot\cdot e_a. \tag{3.9}$$

After summation of Eq. (3.8) we obtain an equation for the change in entropy of the mixture s. It has the form

$$\rho \frac{d^{(a)} s}{dt} + \sum_{k=1}^{n} J_k^a \cdot \nabla s = \sum_{k=1}^{n} \frac{\rho_k h_k^*}{T_k} - \nabla \cdot \sum_{k=1}^{n} \left[\frac{q_k}{T_k} - \frac{J_k^a}{T_k} (\mu_k - F)\right] + \sigma, \tag{3.10}$$

where σ is an expression for the entropy production of the mixture as a whole, satisfying the inequality

$$0 \leqslant \sigma = \sum_{k=1}^{n-1} X_k \frac{T_n - T_k}{T_n T_k} - \sum_{k=1}^{n} \left[q_k - J_k^a \left(\frac{h_k}{c_k} - F\right) + \right.$$
$$\left. + \sum_{i=1}^{n} J_i^a T_k s_k \right] \cdot \frac{\nabla T_k}{T_k^2} + \sum_{k=1}^{n-1} J_k^a \cdot \left[d_k^a - \left(\frac{Q_k^a}{\rho_k T_k} - \frac{a_k \rho_n Q_n^a}{a_n \rho_k T_n}\right)\right] + \frac{1}{T_n} \tau_a \cdot\cdot e_a + \sum_{k=1}^{n} \frac{1}{T_k} R_k^a \cdot\cdot \nabla \frac{J_k^a}{\rho_k}, \tag{3.11}$$

where

$$d_k^a = \left(\frac{f_k}{T_k} - \frac{a_k \rho_n f_n}{a_n \rho_k T_n}\right) - \sum_{i=1}^{n-1} \left(\frac{\delta_{ki}}{T_i} + \frac{a_k \rho_i}{a_n \rho_k T_n}\right) (\nabla \mu_i)_{p, T_i} -$$
$$- \left[- \frac{a_k}{a_n \rho_k T_n} + \sum_{i=1}^{n-1} \left(\frac{\delta_{ik}}{T_i} + \frac{a_k \rho_i}{a_n \rho_k T_n}\right) v_i\right] \nabla p - \left(\frac{1}{T_k} - \frac{a_k \rho_n}{a_n \rho_k T_n}\right) \frac{d^{(a)} u_a}{dt}. \tag{3.12}$$

In Eqs. (3.10)-(3.12) the quantity F denotes free energy of the mixture; h_k, enthalpy; and v_k, the specific partial volume of the k-th component.

Inequality (3.11) represents a generalization of Prigogine's theorem [2, 3] for the general case of motion of homogeneous multitemperature mixtures. It will serve as a basis for the construction of definitive relationships.

The linear definitive relationships satisfying Curie's principle and Onsager's reciprocity relationships [2] have the form

$$X_k = \sum_{i=1}^{n-1} \varkappa_{ki} \frac{T_n - T_i}{T_n T_i} + \gamma_k^a \frac{1}{T_n} (\nabla \cdot u_a) + \sum_{i=1}^{n} \gamma_{ki}^a \frac{1}{T_i} \left(\nabla \cdot \frac{J_i^a}{\rho_i}\right);$$

$$- \left[q_k - J_k^a \left(\frac{h_k}{c_k} - F\right) + \sum_{i=1}^{n} J_i^a T_k s_k\right] =$$

$$= \sum_{i=1}^{n} \eta_{ki}^a \frac{\nabla T_i}{T_i^2} + \sum_{i=1}^{n-1} \nu_{ki}^a \left\{d_i^a - \left(\frac{Q_i^a}{\rho_i T_i} - \frac{a_i \rho_n Q_n^a}{a_n \rho_i T_n}\right)\right\};$$

$$J_k^a = - \sum_{i=1}^{n} \nu_{ki}^a \frac{\nabla T_i}{T_i^2} + \sum_{i=1}^{n-1} \alpha_{ki}^a \left\{d_i^a - \left(\frac{Q_i^a}{\rho_i T_i} - \frac{a_i \rho_n Q_n^a}{a_n \rho_i T_n}\right)\right\};$$

$$\tau_a = \frac{\lambda_a}{T_n}(\nabla\cdot u_a)I + \frac{2\mu_a}{T_n}e_a - \sum_{k=1}^{n-1}\gamma_k^a\frac{T_n - T_k}{T_nT_k}I + \sum_{k=1}^{n}\beta_k^{a1}\frac{1}{T_k}\left(\nabla\cdot\frac{J_k^a}{\rho_k}\right)I + \sum_{k=1}^{n}\beta_k^{a2}\frac{1}{T_k}\left(\nabla\frac{J_k^a}{\rho_k}\right)^c;$$

$$\begin{aligned} R_k^a = {} & \sum_{i=1}^{n}\beta_{ki}^{a1}\frac{1}{T_i}\left(\nabla\cdot\frac{J_i^a}{\rho_i}\right)I + \sum_{i=1}^{n}\beta_{ki}^{a2}\frac{1}{T_i}\left(\nabla\frac{J_i^a}{\rho_i}\right)^c + \\ & + \sum_{i=1}^{n}\beta_{ki}^{a3}\frac{1}{T_i}\left(\nabla\frac{J_i^a}{\rho_i}\right)^A + \beta_k^{a1}\frac{1}{T_n}(\nabla\cdot u_a)I + \beta_k^{a2}\frac{1}{T_n}e_a - \sum_{i=1}^{n}\gamma_{ki}^a\frac{T_n - T_i}{T_nT_i}I; \end{aligned} \tag{3.13}$$

$$\varkappa_{ki} = \varkappa_{ik};\quad \nu_{ki}^a = \nu_{ik}^a;\quad \alpha_{ki}^a = \alpha_{ik}^a;\quad \beta_{ki}^{a1} = \beta_{ik}^{a1};\quad \beta_{ki}^{a2} = \beta_{ik}^{a2};\quad \beta_{ki}^{a3} = \beta_{ik}^{a3}.$$

Moreover, conditions (1.2) and (2.4) impose the following additional limitations on the phenomenological coefficients:

$$\begin{gathered} \sum_{k=1}^{n}\frac{a_k}{\rho_k}\nu_{ki}^a = 0;\quad \sum_{k=1}^{n}\frac{a_k}{\rho_k}\beta_{ki}^{a1} = 0;\quad \sum_{k=1}^{n}\frac{a_k}{\rho_k}\beta_{ki}^{a2} = 0; \\ \sum_{k=1}^{n}\frac{a_k}{\rho_k}\beta_{ki}^{a3} = 0; \\ \sum_{k=1}^{n}\frac{a_k}{\rho_k}\beta_k^{a1} = 0;\quad \sum_{k=1}^{n}\frac{a_k}{\rho_k}\beta_k^{a2} = 0;\quad \sum_{k=1}^{n}\frac{a_k}{\rho_k}\gamma_{ki}^a = 0; \\ \sum_{k=1}^{n}\frac{a_k}{\rho_k}\alpha_{ki}^a = 0. \end{gathered} \tag{3.14}$$

We note that with consideration of Eqs. (2.3) and (2.5), within Eq. (3.13) the expressions for determination of diffusion flows J_k^a are differential equations. Because of their complexity they will not be reproduced here.

If all mixture components have an identical temperature T, then the results obtained in this section are greatly simplified. In this case the equation for change in total energy has the form

$$\rho\frac{d^{(a)}s}{dt} + \sum_{k=1}^{n}J_k^a\cdot\nabla s = \rho\frac{h^*}{T} - \nabla\cdot\left[\frac{q}{T} - \frac{1}{T}\sum_{k=1}^{n}J_k^a\left(\mu_k + \frac{p}{\rho} - \sum_{i=1}^{n}\mu_ic_i\right)\right] + \sigma, \tag{3.15}$$

where

$$\begin{aligned} 0 \leqslant \sigma = {} & -\left(q + \sum_{k=1}^{n}J_k^a\varepsilon\right)\cdot\frac{\nabla T}{T^2} + \sum_{k=1}^{n-1}J_k^a\Big[d_k^a - \\ & - \frac{1}{T}\left(\frac{Q_k^a}{\rho_k} - \frac{a_k\rho_nQ_n^a}{a_n\rho_k}\right)\Big] + \frac{1}{T}\tau_a\cdot\cdot\, e_a + \sum_{k=1}^{n}\frac{1}{T}R_k^a\cdot\cdot\left(\nabla\frac{J_k^a}{\rho_k}\right); \\ d_k^a = {} & \frac{1}{T}\Big\{\left(f_k - \frac{a_k\rho_n}{a_n\rho_k}f_n\right) - \left(1 - \frac{a_k\rho_n}{a_n\rho_k}\right)\frac{d^{(a)}u_a}{dt} - \\ & - \sum_{i=1}^{n-1}\left(\delta_{ki} + \frac{a_k\rho_i}{a_n\rho_k}\right)(\nabla\mu_i)_{p,T} - \left(v_k - \frac{a_k\rho_n}{a_n\rho_k}v_n\right)\nabla p\Big\}. \end{aligned} \tag{3.16}$$

In the case where the terms R_k^a and $\nabla\cdot J_k^aJ_k^a/\rho_k$ may be neglected, we obtain the following equations for determination of heat flux, diffusion flows, and viscosity tensor:

$$-\left(q+\sum_{k=1}^{n} J_k^a \varepsilon\right)=\left[\alpha^a-\sum_{k=1}^{n-1} \alpha_k^a\left(h_k-\frac{a_k\rho_n}{a_n\rho_k}h_n\right)\right]\frac{\nabla T}{T^2}+\sum_{k=1}^{n-1}\alpha_k^a\left[d_k^a-\frac{a_k\rho_n}{a_n^2\rho_k}\sum_{i=1}^{n} J_i^a\frac{d^{(a)}}{dt}\left(\frac{a_i}{\rho_i}\right)-\right.$$

$$\left.-\sum_{i=1}^{n-1}\left(\frac{\delta_{ki}}{\rho_k}+\frac{a_k a_i\rho_n}{\rho_k\rho_i a_n^2}\right)\frac{D^{(a)}J_i^a}{Dt}\right];$$

$$\frac{D^{(a)}J_e^a}{Dt}=-\sum_{j=1}^{n-1}B_{ej}^a\left\{J_j^a+\left[\alpha_j^a-\sum_{k=1}^{n-1}\alpha_{jk}^a\left(h_k-\frac{a_k\rho_n}{a_n\rho_k}h_n\right)\right]\frac{\nabla T}{T^2}\right\}- \tag{3.17}$$

$$-\sum_{k=1}^{n-1}\alpha_{jk}^a\left[d_k^a-\frac{a_k\rho_n}{\rho_k a_n^2}\sum_{i=1}^{n}J_i^a\frac{d^{(a)}}{dt}\left(\frac{a_i}{\rho_i}\right)\right];$$

$$\tau_a=\frac{\lambda_a}{T}(\nabla\cdot u_a)I+\frac{2\mu_a}{T}e_a.$$

Depending on the choice of the normalized weight system a_k, we obtain different equations for determination of heat flux and diffusion flows from Eq. (3.17). If $D^{(a)}J_k^a/Dt$ can be neglected and it is possible to take $a_k = c_k$, we then obtain the classical results [1, 2].

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